MR No. L5L03

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

# WARTIME REPORT

ORIGINALLY ISSUED
December 1945 as
Memorandum Report L5L03

## ANALYTICAL INVESTIGATION OF THE STABILITY OF AN F8F DROPPING MODEL WITH AUTOMATIC STABILIZATION

By Doris Cohen

Langley Memorial Aeronautical Laboratory
Langley Field, Va.



NACA

WASHINGTON



L - 780

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

MEMORANDUM REPORT

for the

Bureau of Aeronautics, Navy Department

# ANALYTICAL INVESTIGATION OF THE STABILITY OF AN F8F DROPPING MODEL WITH AUTOMATIC STABILIZATION

By Doris Cohen

## SUMMARY

Calculations have been made of the stability of a four-tenths scale model of the Grumman F8F airplane, intended for use in dropping tests, under the influence of an automatic pilot. Two types of automatic pilot were considered, one pneumatic and one electric, both causing deflection of the control surfaces at a fixed rate, but differing in the rates available and in lags. Both pilots include a follow-up mechanism designed to give quasi-proportional control. The calculations cover different ratios of control surface deflection to airplane displacement and different rates of control deflection, and investigate the effects of lag, overshoot, and dead band in the servo system.

Recommendations are made as to the control parameters necessary to secure stability of the model. Some general conclusions concerning constant-rate control are also indicated.

## INTRODUCTION

At the request of the Naval Aircraft Experimental Station, calculations have been made of the stability of a four-tenths scale model of the Grumman F8F airplane with automatic pilot, in order to determine the proper rate and proportion of control to be applied.

The F8F is a conventional low-wing single-seater design. The four-tenths scale models are intended to be dropped from considerable altitudes for the purpose of obtaining various aerodynamic and structural data at high speeds. The object of the investigation was to determine follow-up ratios and servo motor rates that would enable the autopilot to stabilize the models in attitude without introducing excessive accelerations. Accuracy in maintaining the original direction of flight was to be considered secondary to the elimination of violent motions which would invalidate or obscure the data obtained from the flight.

Two automatic pilots were under consideration, one pneumatic and one electrical. In each there is a pitch gyro, with spin axis vertical, operating the elevators, and a bank gyro, with its spin axis along the lateral axis of the airplane, operating the ailerons. The electrical pilot includes in addition a directional gyro for operating the rudder. It was planned to use the rudder only if a reasonably straight course in azimuth could not be attained by use of the ailerons alone.

Both the pneumatic and the electric systems provide fixed-rate control deflection governed by a follow-up mechanism designed to give quasi-proportional control; that is, the follow-up system calls for control in proportion to the displacement of the airplane from the reference attitude. This condition is approached, however, only insofar as the fixed rate of control deflection provided by the servo motor is equal to or exceeds the rate of displacement called for by the follow-up. In general this condition is not satisfied and the control is not proportional to the airplane displacement but is applied at a fixed rate.

In action the two systems differ chiefly in the rates of control deflection available and the lags associated with each speed. The calculations described herein cover a range of servo speeds and control gearings and investigate the effect of lags; the resulting conclusions should be applicable to both systems.

All results are presented in the form of motions following abrupt displacements of fixed magnitude - 20° in bank and 15° in pitch and yaw - from the reference attitude. With a nonlinear (nonproportional) control system, an airplane may make a stable recovery from one disturbance but be unable to recover from a larger disturbance. In order to arrive at some sort of conclusions as to a satisfactory system, the ability to recover from any disturbance up to those specified was decided on as the criterion for "stability." In addition, the requirement of small accelerations was imposed and a response was considered satisfactory when no violent motions of the model were involved. Thus a slow return to the reference attitude was not considered particularly objectionable, nor was a long-period hunting motion. No limiting value for the acceleration was specified, but rather it was assumed that recommendations could be made as to methods of minimizing the accelerations and that the designers could then go as far in applying the recommendations as was practical.

## Description of the Automatic Pilot

Schematic diagrams of the two systems considered here are reproduced in figures 1 and 2. There is no essential difference between the components of each system - elevator, aileron, and in the electrical pilot, rudder control. However, each is separately adjustable to give the desired stabilization in the related degree of freedom. The main features of the automatic pilots, as they enter into the mathematical analysis, will be reviewed here.

A deviation of the airplane from the gyroscope reference causes a cut-off plate to move between a jet and orifice fixed on the gyroscope. Although the signal pressure thus transmitted is approximately proportional to the angular deviation of the airplane, it acts only to open or close a double-throw electric switch, causing the servo motor to operate in one direction or the other. The action involves closing a gap of finite magnitude in making contact and a certain minimum of signal pressure is therefore required before any control is applied. This minimum corresponds to a certain small angular error, or dead band, through which the airplane may deviate from the reference without causing any activation of the servo motor. The dead band is determined chiefly by the distance between the contacts in the pneumatic-electric pick-off and is adjustable.

A certain amount of control-surface motion, however, does take place while the pick-off points are in the dead band; this motion is the result of coasting of the servo motor. Because of it, a lower limit for the width of the dead band has been specified for each follow-up pulley and servo motor rate, to prevent a self-excited hunting of the servo system.

When the error in heading exceeds the magnitude of the dead band, the electrical contact is closed and the servo motor causes a linear member, or rod, to take up motion in one direction or the other at a fixed rate. The rod is ultimately geared to the control surface.

A follow-up cable attached to the rod causes rotation of the cut-off valve, or gyro reference, so that the reference heading differs from the original heading by an amount proportional to the control deflection. Conversely, as long as the airplane heading is such that the signal is zero, the control deflection is proportional to the deviation in heading from the original. The factor of proportionality is dependent on the diameter of the follow-up pulley and the linkage between the servo motor and the control surface. It should be noted that since the control surface, and therefore the gyro reference, moves at a fixed rate, proportional control is not actually maintained while the airplane heading is changing.

Parameters for use in analysis.- The pneumatic servo motor offers a range of piston speeds from 0.19 to 6.5 inches per second, with the use of the lowest part of the range not recommended because of the excessive lag in operation. The piston has a 4-inch travel and the actual speed of deflection of the control surface depends on the total deflection to which that 4 inches corresponds, or the gearing between the control surface and the piston. It will be convenient to denote this gearing ratio by $K_1 = \frac{1}{4} \times$ total deflection (hard over to hard over), or degrees of deflection per inch of piston displacement. The rate of control surface deflection $\dot{\delta}$ will than equal $K_1$ times the piston speed. The ratio of control deflection to airplane displacement, K, will be the product of $K_1$ and the ratio $K_2$ giving the number of inches of piston movement corresponding to one degree of pick-off displacement. The follow-up pulleys provided with the original mechanism were standard-diameter pulleys that gave values of $K_2$ equal to 0.09, 0.065 and 0.04 inch per degree. It was planned to provide 44° of aileron deflection and 50° of elevator. Presumably the rudder would likewise be geared to provide 50° total deflection. Then the value of $K_1$ for the ailerons would be 11° per inch and for elevator and rudder, 12.5° per inch. The ranges of $\dot{\delta}$ and K provided by the pneumatic servo with the gearings described are therefore as follows:

TABLE I

| Pulley diameter (in.) | $K_2$ (in./deg) | Ailerons ($K_1$ = 11° per in.) | | Elevator and rudder ($K_1$ = 12.5° per in.) | |
|---|---|---|---|---|---|
| | | *K | $\dot{\delta}_a$ (deg/sec) | *K | $\dot{\delta}$ (deg/sec) |
| 1/2 | 0.04 | 0.44 | 2.1 to 71.5 | 0.5 | 2.4 to 81.2 |
| 3/4 | .065 | .72 | 2.1 to 71.5 | .81 | 2.4 to 81.2 |
| 1 | .09 | .99 | 2.1 to 71.5 | 1.13 | 2.4 to 81.2 |

*The values of K are based on the measured value of $K_2$. They were intended to be approximately 1/2, 3/4, and 1/1.

The electric motor is essentially the same as the pneumatic one, with a 4-inch linear motion to be transmitted to the aileron horn. There are, however, only three servo speeds available: 0.57, 0.88 and 2.35 inches per second. The minimum dead band (in deg) determined experimentally for each servo condition is given in table II:

TABLE II

| Pulley diameter (in.) | Pneumatic servo | | | Electric servo | | |
|---|---|---|---|---|---|---|
| | Approximate servo speed, in./sec | | | | | |
| | 0.19 | 0.37 | 6.5 | 0.57 | 0.88 | 2.35 |
| 1/2 | 0.3 | 0.4 | 1.5 | 0.5 | 0.4 | 2.2 |
| 3/4 | Not measured | | | | .5 | 1.7 |
| 1 | .2 | .3 | 1.5 | .2 | .3 | 1.0 |

<u>Lag and servo coast.</u>- Two other quantities entering into the calculations are the time lag between reversal of signal and reversal of control motion and the amount of coast. These are difficult to ascertain exactly because they always occur in conjunction with the time occupied in passage of the pick-off through the dead band. They also vary considerably according to the existing pressure level of the system, depending on the direction and duration of the previous signal. As data on these characteristics were not available when the computations were begun, the initial calculations made on the General Electric differential analyzer omitted them entirely. For later calculations the experimentally determined lag data tabulated in the following table were used:

TABLE III

| Pulley diameter (in.) | Pneumatic servo | | | Electric servo | | |
|---|---|---|---|---|---|---|
| | Approximate servo speed, in./sec | | | | | |
| | 0.19 | 0.37 | 6.5 | 0.57 | 0.88 | 2.35 |
| 1/2 | 1.5 | 0.8 | 0.17 | 0.1 | 0.08 | 0.1 |
| 3/4 | Not measured | | | | .05 | .1 |
| 1 | 1.7 | .85 | .17 | .1 | .10 | .1 |

The amount of coast was estimated from the dead band necessary to prevent hunting of the servo system in each condition. These data are presented in figure 3.

## DESCRIPTION OF COMPUTATIONS

### Equations of Motion

The calculations were based on the linearized equations of motion of the airplane, used in the following form:

Longitudinal

$$\left\{\begin{aligned}
&\frac{\mu b}{V}\frac{du}{dt} = -C_D u - \frac{1}{2}\left(C_{D_\alpha} - \frac{W\cos\gamma_o}{qS}\right)\alpha - \frac{W\cos\gamma_o}{2qS}\theta \\
&\frac{\mu b}{V}\left(\frac{d\theta}{dt} - \frac{d\alpha}{dt}\right) = \left(\frac{C_{L_\alpha}}{2} - \frac{W\sin\gamma_o}{2qS}\right)\alpha + C_L u + \frac{W\sin\gamma_o}{2qS}\theta \\
&\frac{2A\mu K_y^2}{V^2}\frac{d^2\theta}{dt^2} = C_{m_{\dot\alpha}}\frac{d\alpha}{dt} + C_{m_\alpha}\alpha + C_{m_{\dot\theta}}\frac{d\theta}{dt} + C_{m_{\delta_e}}\delta_e
\end{aligned}\right.$$

Lateral

$$\left\{\begin{aligned}
&\frac{2\mu b}{V}\left(\frac{d\beta}{dt} + \frac{d\psi}{dt}\right) = C_{y_\beta}\beta + \frac{W}{qS}\cos\gamma_o\,\phi + \frac{W}{qS}\sin\gamma_o\,\psi \\
&\frac{2\mu K_z^2}{V^2}\frac{d^2\psi}{dt^2} = C_{n_\beta}\beta + C_{n_{\dot\phi}}\frac{d\phi}{dt} + C_{n_{\dot\psi}}\frac{d\psi}{dt} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r \\
&\frac{2\mu K_x^2}{V^2}\frac{d^2\phi}{dt^2} = C_{l_\beta}\beta + C_{l_{\dot\phi}}\frac{d\phi}{dt} + C_{l_{\dot\psi}}\frac{d\psi}{dt} + C_{l_{\delta_a}}\delta_a
\end{aligned}\right.$$

The symbols used in the equations are defined and their values in high-speed and low-speed flight are given in the following section (table IV).

## Flight Conditions and Numerical Values Assumed

## for the Calculations

A one-fifth scale model for the airplane had been tested in the Langley atmospheric wind tunnel, and the lift, drag, and moment data required for the stability calculations were, with a few exceptions, obtained from these tests. Theoretical corrections were applied for the effect of the propeller, with which the models were tested but which will not be present in the drop tests.

It was assumed that the model would be released in level flight at zero lift at about 200 miles per hour, and that the pick-off in the pitch gyro would be trimmed over to stabilize the model along a $30^{\circ}$ glide path, which is the angle for equilibrium flight at a Mach number of 0.8 for this airplane. The bank gyro was assumed to measure angular deviation about this glide path. The quantities tabulated represent the conditions at the beginning of the glide (200 miles per hour) and at the equilibrium or terminal speed, 580 miles per hour.

TABLE IV

| Symbol | Definition | High speed | Low speed |
|---|---|---|---|
| $\gamma_0$ | angle of climb | $-30^{\circ}$ | $-30^{\circ}$ |
| V | speed along flight path | 850 fps | 300 fps |
| A | aspect ratio of wing | 5.17 | 5.17 |
| b | span of wing | 14.2 ft | 14.2 ft |
| S | area of wing | 39.0 sq ft | 39.0 sq ft |
| $\mu$ | $\frac{1}{\rho Sb} \times$ mass of airplane | 66.4 | 114.3 |
| $\rho$ | density of air, corresponding to alititudes of | 0.001267<br>20,000 ft | 0.000736<br>35,000 ft |
| q | dynamic pressure = $\frac{1}{2}\rho V^2$ | 458 | 33.1 |
| W | weight of airplane | 1500 lb | 1500 lb |
| $K_y$ | radius of gyration about lateral axis | 2.965 ft | 2.965 ft |

TABLE IV - Continued

| Symbol | Definition | High speed | Low speed |
|---|---|---|---|
| $u$ | fractional departure from mean forward speed = $\frac{\Delta V}{V}$ | Variable | |
| $\alpha$ | departure from mean angle of attack, radians | Variable | |
| $\theta$ | departure from mean (gyro reference) attitude, radians | Variable | |
| $\delta_e$ | angle of elevator deflection (positive down) measured from the trim position | Variable | |
| $C_D$ | drag coefficient | 0.046 | 0.026 |
| $C_{D_\alpha}$ | rate of change of drag coefficient with angle of attack $\partial C_D/\partial\alpha$ | 0 | 0 |
| $C_L$ | mean lift coefficient | 0.08 | 0 |
| $C_{L_\alpha}$ | rate of change of lift coefficient with angle of attack, $\partial C_L/\partial\alpha$, per radian | 4.2 | 4.2 |
| $C_{m_\alpha}$ | pitching-moment coefficient due to unit change in $\alpha$, per radian | -0.422 | -0.422 |
| $C_{m_{\dot\alpha}}$ | pitching-moment coefficient due to unit rate of change of $\alpha$, per radian per second | -0.00794 | -0.0225 |
| $C_{m_{\dot\theta}}$ | pitching-moment coefficient due to unit rate of change of $\theta$, per radian per second | -0.0141 | -0.0400 |

TABLE IV - Continued

| Symbol | Definition | High speed | Low speed |
|---|---|---|---|
| $C_{m_{\delta_e}}$ | pitching-moment coefficient due to unit deflection of the elevator | -0.716 | -0.716 |
| $\beta$ | angle of sideslip | Variables | |
| $\phi$ | angle of bank | Variables | |
| $\psi$ | angle of yaw | Variables | |
| $K_z$ | radius of gyration about vertical axis | 3.02 ft | 3.02 ft |
| $K_x$ | radius of gyration about longitudinal axis | 1.60 ft | 1.60 ft |
| $\delta_a$ | angle of aileron deflection (positive when right aileron is down) measured from the trim position | Variable | |
| $\delta_r$ | angle of rudder deflection (positive when trailing edge is to the left of hinge) | Variable | |
| $C_{y_\beta}$ | sideforce coefficient due to unit angle of sideslip, per radian | -0.395 | -0.401 |
| $C_{n_\beta}$ | yawing-moment coefficient due to unit angle of sideslip, per radian | 0.0736 | 0.0754 |
| $C_{n_{\dot{\phi}}}$ | yawing-moment coefficient due to unit rate of rolling, per radian per second | -0.0000187 | 0 |
| $C_{n_{\dot{\psi}}}$ | yawing-moment coefficient due to unit rate of yawing, per radian per second | -0.000894 | -0.00582 |

TABLE IV - Concluded

| Symbol | Definition | High speed | Low speed |
|---|---|---|---|
| $C_{n_{\delta_a}}$ | yawing-moment coefficient due to unit aileron deflection, per radian | 0 | 0 |
| $C_{n_{\delta_r}}$ | yawing-moment coefficient due to unit rudder deflection, per radian | -0.0453 | -0.0453 |
| $C_{l_\beta}$ | rolling-moment coefficient due to unit angle of sideslip, per radian | -0.0620 | -0.0596 |
| $C_{l_{\dot\phi}}$ | rolling-moment coefficient due to unit rate of rolling, per radian per second | -0.00309 | -0.0211 |
| $C_{l_{\dot\psi}}$ | rolling-moment coefficient due to unit rate of yawing, per radian per second | -0.000167 | 0 |
| $C_{l_{\delta_a}}$ | rolling-moment coefficient due to unit aileron deflection, per radian deflection of each aileron | -0.146 | -0.146 |

## Method of Calculating the Motions

The larger part of the calculations was carried out on the General Electric differential analyzer. (For a description of the analyzer, see reference 1.) As has been stated, these calculations omitted the consideration of either time lag or servo coast. They showed, however, the effect of the other parameters $\dot\delta$, K, and the dead band.

The relations set up on the differential analyzer were the equations of motion and the two additional relations $\delta = +\dot\delta t$ and $\delta = -\dot\delta t$, where by $\dot\delta$ is meant the absolute magnitude of the rate of control-surface deflection. The quantity $(\theta - \delta/K)$, representing the deviation of the airplane heading from the adjusted reference (that is, the reference as determined by the control-surface deflection and follow-up), appeared on a counter throughout the calculations. When $|\theta - \delta/K|$ exceeded the magnitude of the dead band, $\delta$ was permitted to change in the proper direction to reduce $|\theta - \delta/K|$. While $|\theta - \delta/K|$ was within the dead band, $\delta$ was kept constant. The calculations were initiated by assuming a value of $\theta$ different from zero, as described in the introduction, thereby causing the application of control.

The method used in the supplementary step-by-step calculations made to determine the effect of lag is somewhat different from that just described. In this case the calculations are based on the response of the airplane to continued displacement of the control surface at the rate $\dot{\delta}$ degrees per second; this response, denoted by $\theta_1$, may be calculated by the method of operators. Plots of $\theta = \theta_0 + \theta_1$ and $\delta/K$ are superimposed (see fig. 4) and the point where $\delta/K$ enters the dead band (shaded portion of fig. 4) with respect to $\theta$ is determined. Allowance can then be made for the coasting of the control through an additional degree or fraction of a degree. When the control surface is considered to have stopped moving (at time $t_1$), the negative of the unit response is added to the $\theta$ thus far obtained, taking as $t = 0$ the instant $t_1$ at which control-surface motion ceases. A second negative response is added beginning at the instant $t_2$ when the control motion starts to reverse. In determining $t_2$ both the dead band and the lag in the servo motor itself can be taken into account. The displacement of the airplane at any instant $t$ is finally

$$\theta = \theta_0 + \theta_1 (t) - \theta_1 (t - t_1) - \theta_1 (t - t_2) + \theta_1 (t - t_3)$$

$$+ \theta_1 (t - t_4) \text{ etc.}$$

if the signal is reversed in direction each time the pick-off emerges from the dead band.

## PRESENTATION AND DISCUSSION OF RESULTS

### Lateral Control (Ailerons Only)

Effects of $K$ and $\dot{\delta}_a$.- As previously explained, the first calculations were made to determine the effect of the ratio $K$ and the rate $\dot{\delta}_a$ on the recovery of the airplane following an abrupt displacement in bank with lag and coast neglected. The results are shown in the following series of figures:

Figure 5. $K = 1{:}2$, $\dot{\delta}_a = 1.5^\circ$, $3^\circ$ and $12^\circ$ per second

Figure 6. $K = 1{:}4$, $\dot{\delta}_a = 0.75^\circ$ and $1.5^\circ$ per second

Figure 7. $K = 1{:}8$, $\dot{\delta}_a = 0.375^\circ$, $0.75^\circ$ and $3^\circ$ per second

In these calculations, the pulley ratio $K_2$ was assumed to be held fixed and ratio $K$ reduced by reduction of $K_1$. Three servo speeds were selected. The reduction of $K_1$ accounts for the reduced values of $\dot{\delta}_a$ used with the smaller control gearings. The dead band was in each case assumed to be the minimum with which to prevent self-excited oscillation of the autopilot system. The values taken assumed the so-called 1:1 pulley to have been used.

It is evident that if $K = 1/2$ or $1/4$ (figs. 5 and 6) the motion following a $20^\circ$ displacement will be unstable and that any increase in $\dot{\delta}_a$, within the range available, would only increase the instability. In these cases, the aileron motion is by no means proportional to the displacement it is intended to correct. (See fig. 6.) Further reduction of the gearing to $1/8$ (fig. 7) improves the phase relation somewhat and results in a stable motion; increasing $\dot{\delta}_a$ at this point results in still further improvement, with the ailerons very nearly in phase with the airplane motion when $\dot{\delta}_a = 3^\circ$ per second. (Compare this motion, however, with that of fig. 5, where $\dot{\delta}_a$ is also equal to $3^\circ$ per second, but $K = 1/2$.)

Effect of the dead band.- Where the motion was stabilized, as in the cases of fig. 7, the final motion (shown only for $\dot{\delta}_a = 3^\circ$ per second) was a steady hunting of amplitude only slightly larger than that of the dead band. This hunting was not considered to be undesirably large, in view of the object of the flights. Nevertheless, the dead band would ordinarily be kept to a minimum in order to minimize the hunting. In this case, however, it was thought that, since the dead band introduced a certain amount of lead into the signal, increasing the dead band above the minimum would be a method of improving the stability. In figure 8, the unstable motion of figure 5 for $\dot{\delta}_a = 1.5^\circ$ per second is compared with the motion resulting from increasing the dead band by a factor of 10. It is seen that there is, in fact, a stabilizing effect, but it is so slight that for any reasonable increase in dead band the effect would be negligible.

Effect of coast and lag.- On the basis of the analyzer calculations, a follow-up of 1/8 and rate of aileron deflection of $3^\circ$ per second were tentatively selected for recommendation.

This condition was then checked (fig. 9) with $0.11^\circ$ of aileron coast, without lag and with 0.15 second lag. The values of coast and lag represent the maximum values for the pneumatic and electric systems using the follow-up ratio and aileron motion rate under consideration. (See fig. 3.)

The dead band was assumed to be $\pm 0.44^\circ$ of error in $\psi$, in order that the servo coast would not carry the pick-off through it.

It is seen that although the stability is reduced about 20 percent by the coast of the servo motor, the motion, with no time lag, is still satisfactory. A time lag as small as 0.15 second, however, causes the control motion to be once again $90^\circ$ out of phase with $\phi$ and results in instability. An intermediate time lag of 0.05 second or more would probably also be unsatisfactory.

On the basis of the previous results it appeared necessary to reduce the ratio K still further. In figure 10 are shown the motions resulting from reducing the value of K to 1/12. In one case the rate of deflection of the ailerons was assumed to be unchanged. In the second case, the value of $\dot{\delta}_a$ was doubled. The reduction in K proved sufficient to offset the destabilizing effect of the lag. As before, with stability secured by the reduction of follow-up ratio, an increase in $\dot{\delta}_a$ resulted in further improvement of the motion. This improvement was obtained in spite of the fact that the amount of coasting was also assumed to have doubled.

In figure 10 is shown also the motion calculated for the low-speed condition with $K = 1/12$ and $\dot{\delta}_a = 3^\circ$ per second, $1^\circ$ of coast and 0.15 second lag. The motion is seen to be entirely satisfactory.

Application of results.- As a result of the calculations it is concluded that the control ratio should not exceed $1^\circ$ of aileron to $12^\circ$ of displacement in bank. The servo-motor speed should be such that the ailerons are deflected at least $3^\circ$ a second. Higher rates of aileron deflection are preferable if the servo system can be made to perform satisfactorily.

From an examination of table I, it is apparent that the aileron system as designed does not include the low value of K necessary for stability in roll. Since $K = K_1K_2$, the necessary reduction in K could be made either in the follow-up stage or in the gearing of the ailerons to the piston. The latter change would, of course, reduce the rate of aileron deflection available, but this amount is already far more than is needed. The total aileron travel would also be reduced.

## Longitudinal Control

Effects of $K_1$ and $\dot{\delta}_e$.- The investigation into the problem of longitudinal control followed the same general program as in the lateral case, except that, because it was possible to specify the total amount of elevator deflection that could be used and might be needed for the contemplated maneuvers, the gearing $K_1$ of elevator to piston was selected and fixed, and changes in the ratio of $\delta_2$ to $\theta$ were made in the follow-up stage. Thus any one servo speed corresponded to the same elevator speed throughout the calculations. The following conditions were investigated in the preliminary calculations:

Figure 11. $K = 1/2$, $\dot{\delta}_e = 11.50^\circ$ and $1.67^\circ$ per second

Figure 12. $K = 1/4$, $\dot{\delta}_e = 6.75^\circ$, $1.67^\circ$ and $0.83^\circ$ per second

Figure 13. $K = 1/8$, $\dot{\delta}_e = 6.75^\circ$ per second

The curves shown are the recoveries from an initial disturbance in pitch of $15^\circ$. As in the case of the ailerons the dead band was adjusted in accordance with table II to the minimum for avoiding self-oscillation in the autopilot mechanism.

The results of the calculations omitting lag, shown in figures 11 to 13, indicate that, unlike the lateral motion, the longitudinal motion is not critically affected by the follow-up ratio in the range investigated. A more significant factor is the rate of elevator deflection which, again unlike the lateral case, is required to be kept small in order to avoid the high accelerations associated with the short-period oscillations. It should be noted that the stability is not a problem in this case. From an examination of the results, it was concluded that a rate of control deflection of $\dot{\delta}_e = 2.5^\circ$ per second would not lead to excessive accelerations, while providing, at the same time, sufficient control in the pull-ups.

Motions with lag.- As shown in table I, the pneumatic servo motor, with the contemplated elevator gearing, permitted a rate of elevator deflection of $2.5^\circ$ per second providing the servo motor was adjusted to its lowest speed. Such an adjustment involves the introduction of large time lags, as described in table III.

Using the lag of 1.5 seconds given in table III for the combination of the smallest pulley and lowest speed, the motion was calculated for $K = 1/2$ and $\dot{\delta}_e = 2.5^\circ$ per second and the results (fig. 14(a)) indicated that such a condition would not be satisfactory. With the lag reduced to 0.5 second, the motion would be fairly well damped at high speeds (fig. 14(b)), but at the beginning of the drop (fig. 14(c)) damping would still be poor.

A better motion, from the point of view of the accelerations involved, would be obtained by decreasing $K$ to 1/5 (fig. 15(a)). Such a reduction in $K$ is effective in improving the motion even if a reduction in lag is not possible (fig. 15(b)). Check calculations at high speed with $K = 1/5$ (fig. 16) show the recovery to be satisfactory with either 0.5 or 1.5 seconds lag.

Application of results.- On the basis of the preceding considerations the recommendations for longitudinal control are as follows:

The rate of elevator deflection should not be over 2.5° per second. If the servo motor speed or the gearing of servo motor to elevator can be reduced so as to reduce $\dot{\delta}_e$ below 2.5° per second, and if the test maneuvers contemplated do not require so rapid a control movement, a lower value of $\dot{\delta}_e$ should be used. The ratio of elevator angle to displacement in pitch sould be of the order of 1/5 or less. By a reduction of the control gearing $K_1$, it is possible to obtain any desired reduction of $K$. At the same time the rate $\dot{\delta}_e$ will be reduced, or alternatively, a higher servo motor speed may be used while maintaining the same rate of elevator deflection. Either modification will improve the motion of the airplane. It should be noted that an elevator motion slower than 2.5° per second cannot be obtained with the electric servo without reducing $K$ to a value less than 1/5.

The reduction of $K_2$ will reduce the total elevator. A reasonable reduction of elevator travel will not be serious as the wind-tunnel tests have indicated that an up travel of 10° from neutral will be ample to stall the model.

## Directional Control

Aileron control.- A certain amount of directional stabilization is obtained in a dive if the airplane is constrained in roll, as by an effective automatic pilot. With the bank stabilization system recommended herein, this incidental stabilization amounts to approximately 0.15° per second of recovery for each degree of deviation in azimuth.

If the inner gimbal of the gyroscope is initially alined for horizontal flight, the subsequent dive of the airplane model will cause the reference to be tilted in such a way as to transmit signals of error in yaw as well as in bank, thus giving an additional degree of stabilization in azimuth. The motion under these conditions following a $15^\circ$ disturbance in yaw was calculated for the assumed $30^\circ$ glide path and is shown in figure 17. It is seen that the stabilizing effect, although larger than in the preceding case, is still small.

For both cases, it should be noted, the stabilizing effect exists only in the dive; the effect will be destabilizing during a climbing maneuver.

Rudder control.- It appears necessary to use the yaw gyro in rudder control in order to secure a positive directional sense. In the calculations for the rudder control, the model was assumed to be prevented from rolling by the aileron system and the equations for lateral motion were solved with only 2 degrees of freedom - yaw and sideslip. Because of the similarity between the lateral motion under this assumption and the longitudinal motion, the control ratio $K$ and the rate of rudder deflection $\dot{\delta}_r$ investigated were those found most favorable for the elevator; that is, $K = 1/5$, $\dot{\delta}_r = 2.5^\circ$ per second. The lag assumed was 0.5 second.

The recovery from $15^\circ$ of error in yaw with these conditions is shown in figure 17. The recovery is reasonably smooth but probably undesirably slow. The rate of recovery was increased somewhat by increasing the ratio $K$ to 1/2 (see fig. 17). The effect of $K$ on the oscillatory part of the motion appears negligible.

Increasing the rate $\dot{\delta}_r$ (fig. 18) is not effective in increasing the rate of recovery of the model, but acts only to increase the amplitude of the oscillations and may even cause instability. Another interesting point indicated by the motions of figure 18 is that, at the particular rudder speed being considered, a reduction of the lag from 0.5 to 0.3 second has an adverse effect on the hunting.

Application of results.- The results of the investigation of directional control indicate that when the model is diving the ailerons will provide a small but definite amount of directional stability but will cause a similar degree of instability when the model is climbing. If the rudder and directional gyro are used for azimuth control a ratio of rudder angle to yaw deviation greater than 1/2 should be provided in order to obtain a fairly rapid return to the reference heading following a disturbance. Deflection of the rudder should not be at a rate higher than $2.5^\circ$ per second.

## CONCLUDING REMARKS

It is desired to point out again that the control system investigated herein does not give proportional control and the results of simplified calculations assuming such control do not apply. The control gearing ratio determines, in this case, not so much the ratio of control deflection to airplane displacement, as the extent to which these quantities are proportional at all. The resulting phase relations determine in a large measure the stability of the autopilot-airplane system.

The specific recommendations of this report are based on the calculated recovery from disturbances of a certain magnitude. Because of the nonproportional nature of the control, the character of the recovery changes with the magnitude of the disturbance. Thus for every recommended condition there will exist some disturbance, greater than the one assumed herein, from which the airplane will not recover. Also, the linearized equations take no account of the possibility of stalling the airplane, a source of some difficulty with previously tested pilotless aircraft. The model should be launched, therefore, as nearly as possible in the equilibrium condition so that the initial disturbance for which the autopilot is called on to correct is kept to a minimum.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

## REFERENCE

1. Kuehni, H. P., and Peterson, H. A.: A New Differential Analyzer. Elec. Eng., AIEE, vol. 63, no. 5, May 1944, pp. 221-228.

[illegible]

## [illegible]

[illegible]

[illegible]

[illegible]

## [illegible]

[illegible]

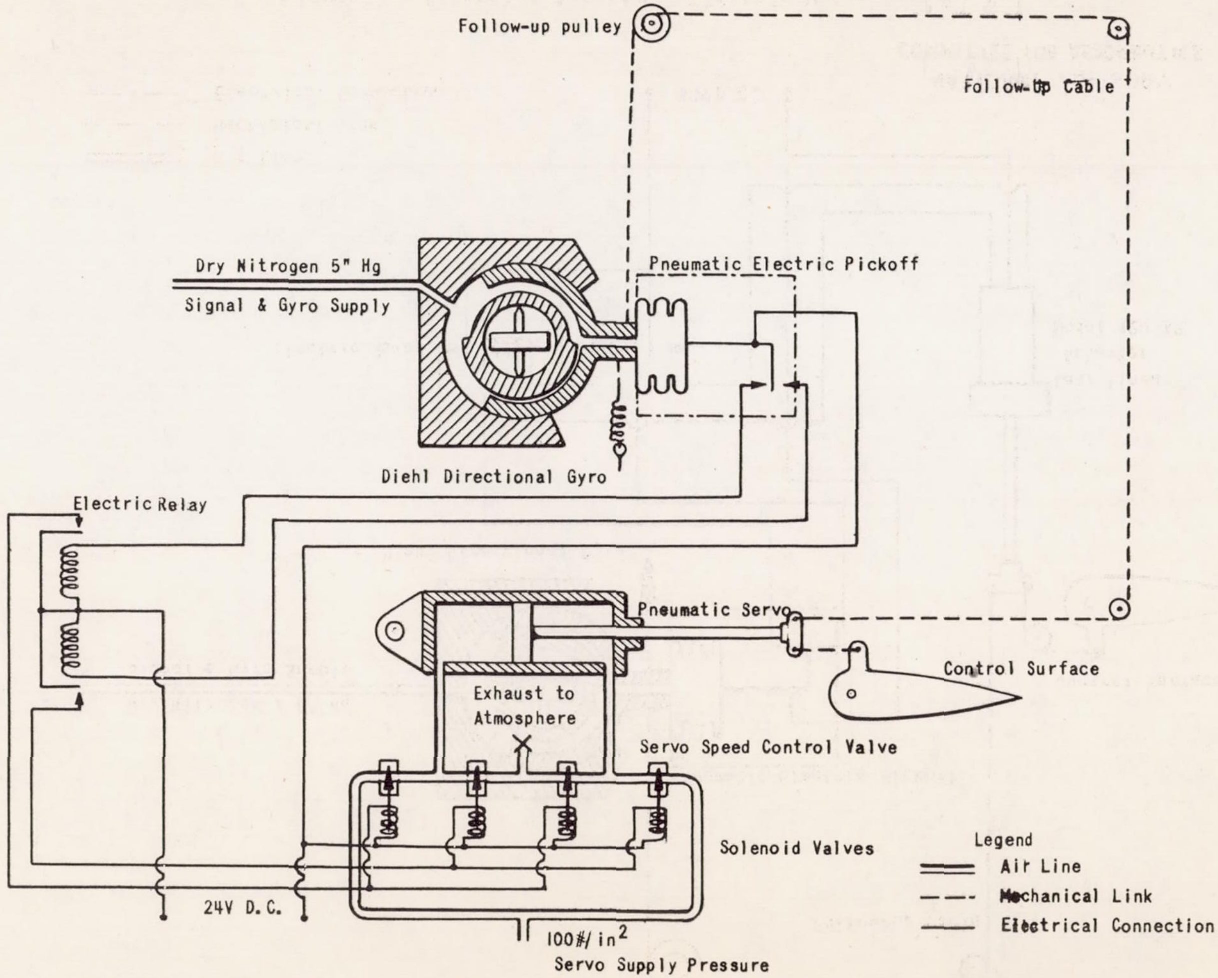
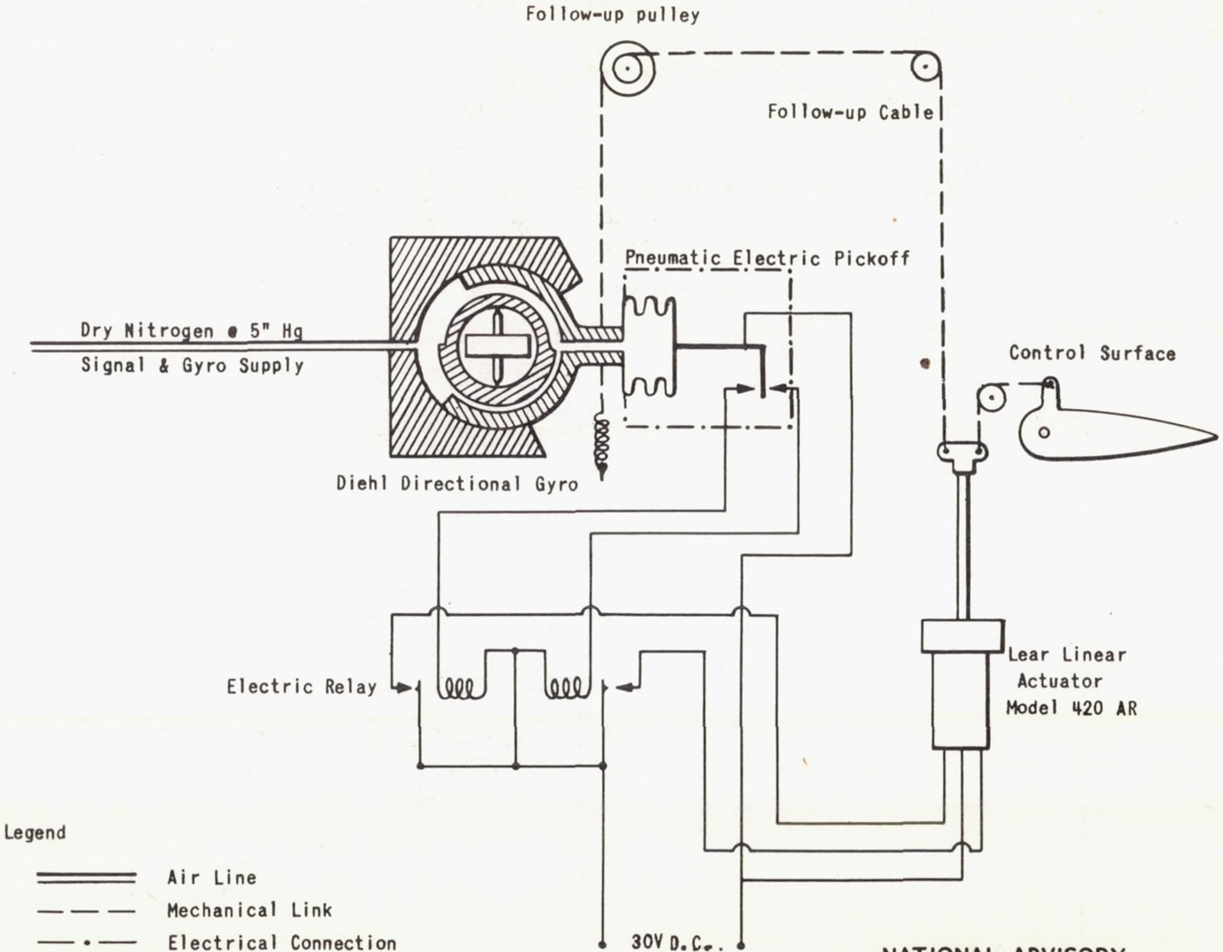



Figure 1.- Schematic Diagram - Pneumatic Servo

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

MR No. L5L03




NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 2.- Schematic Diagram - Electric Motor Servo

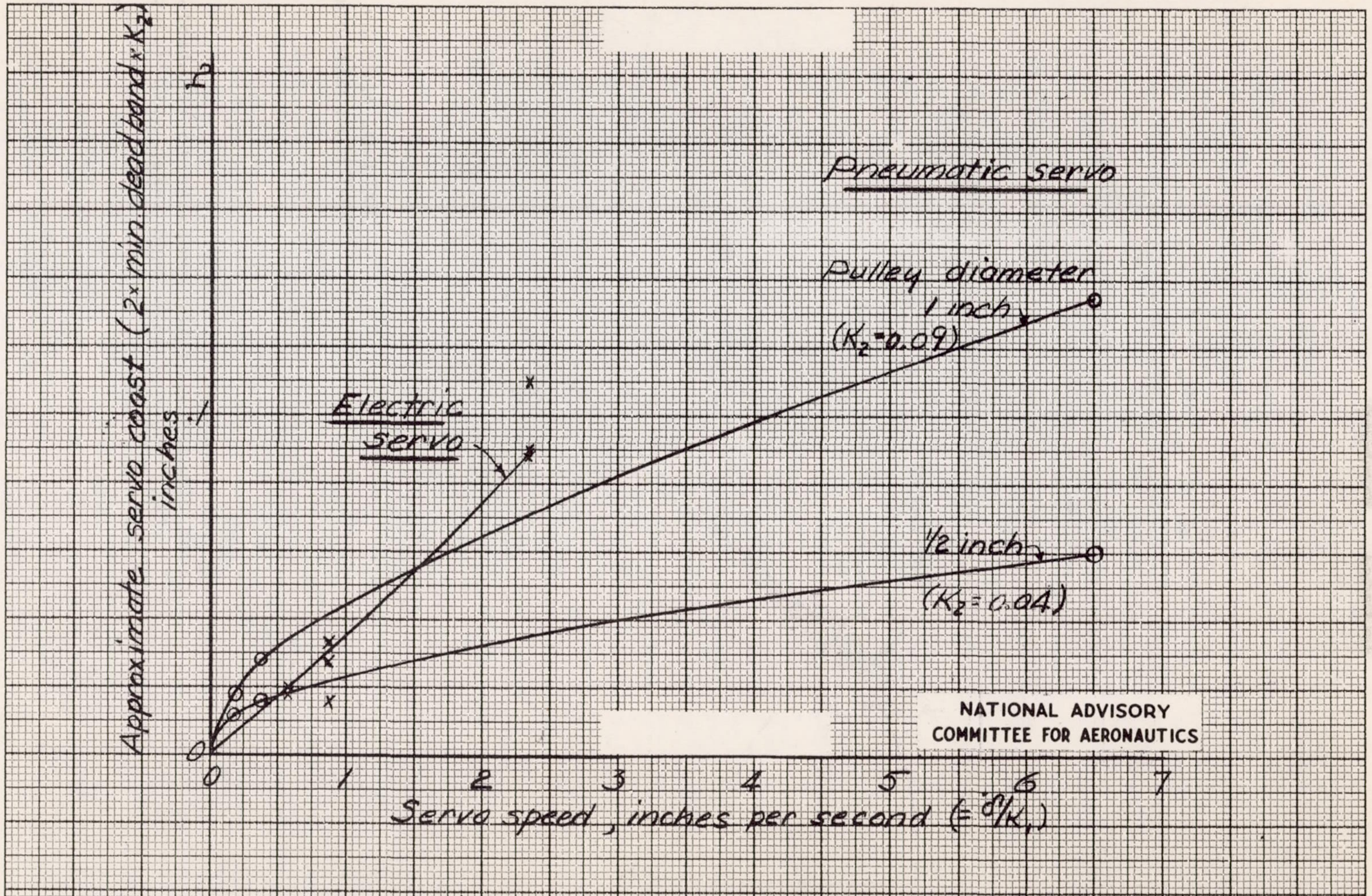



Figure 3.- Approximate coasting of servo motor after removal of the signal.

MR No. L5L03

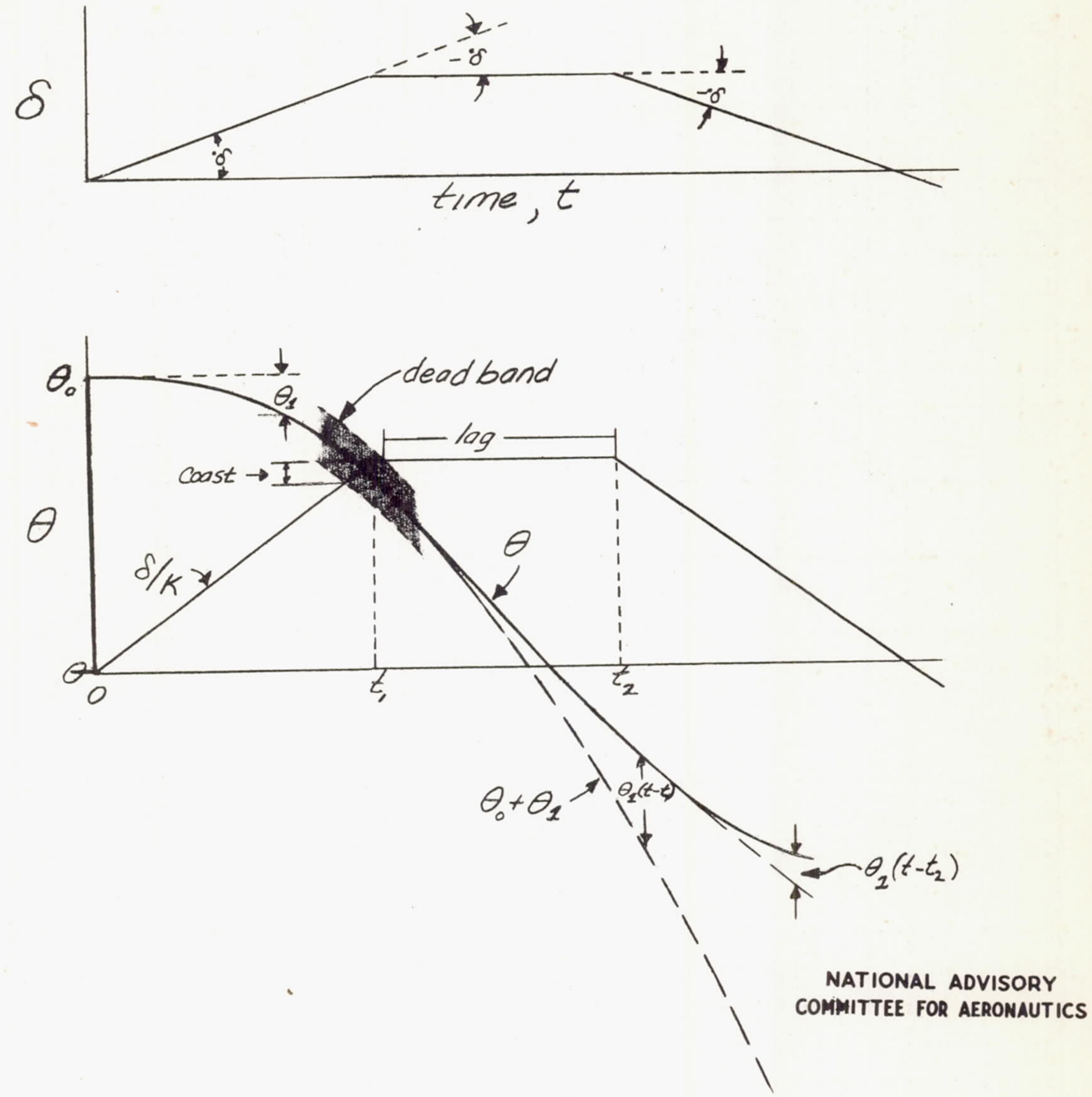
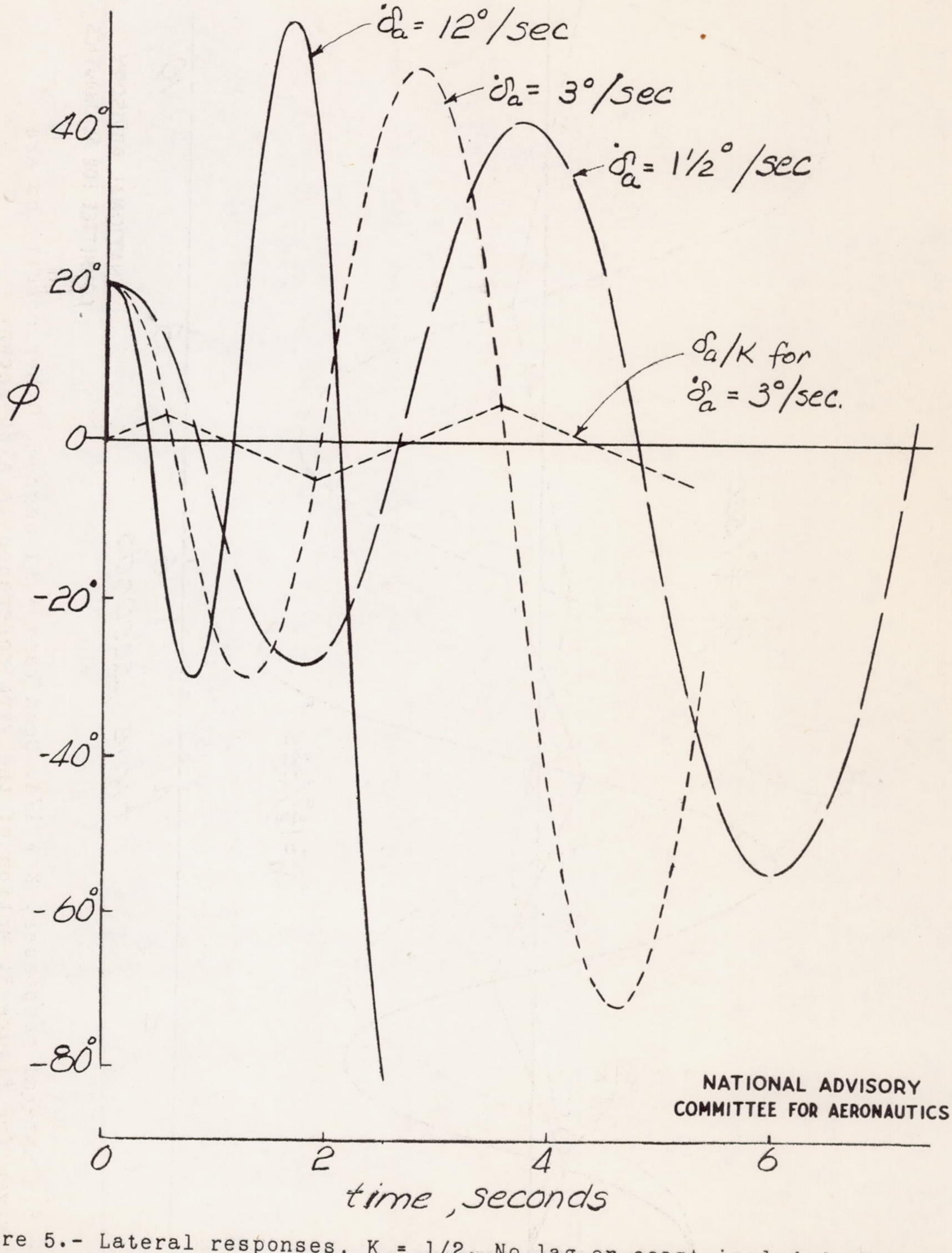
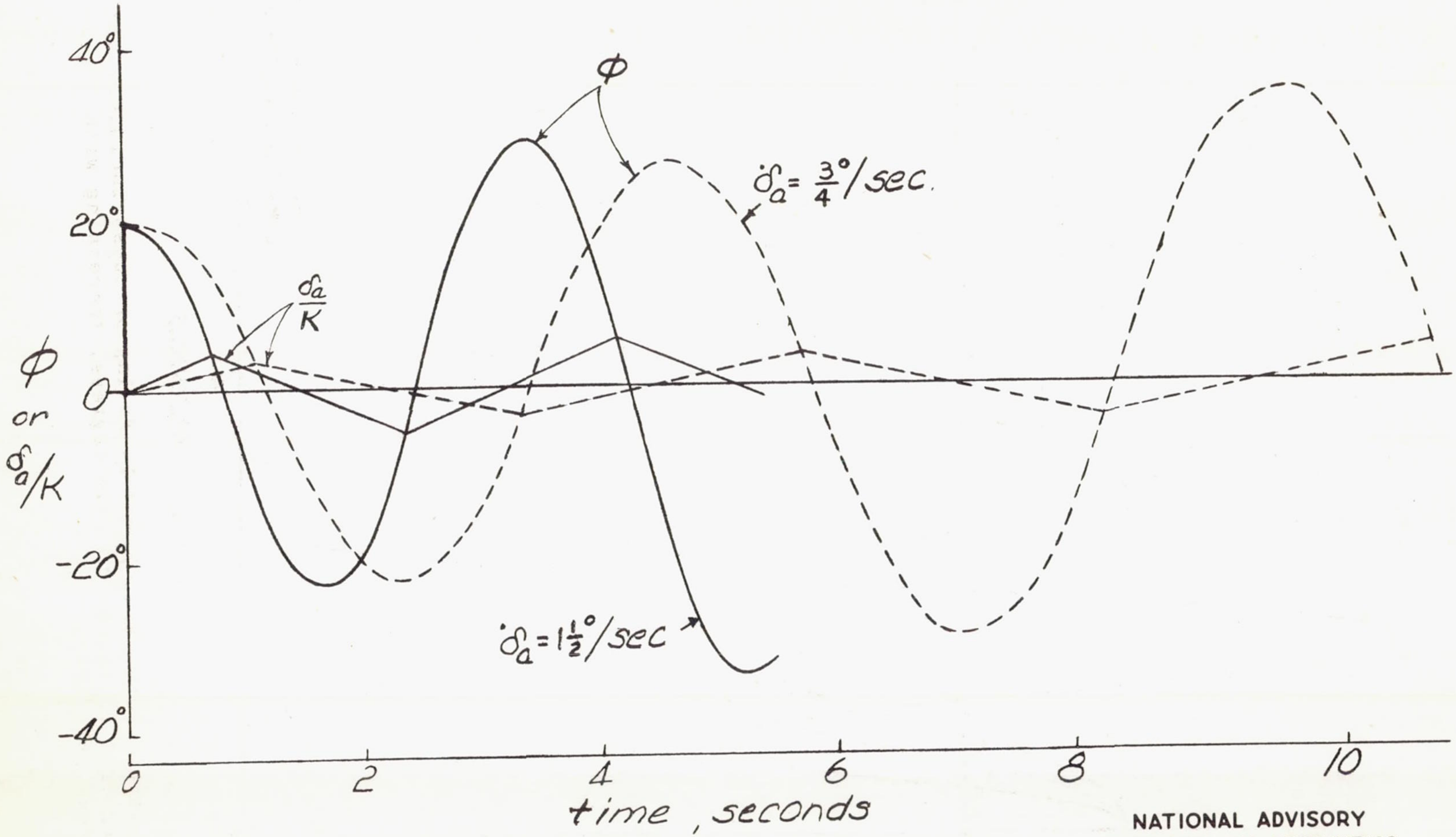



Figure 4.- Semi-graphical method of constructing the response to constant-rate control deflection, with coast, lag and dead band taken into account.




NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Figure 5.- Lateral responses, K = 1/2. No lag or coast included. Dead bands vary from ±1 degree to ±4.5 degrees, increasing with $\dot{\delta}_a$. V = 850 feet per second.




NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 6.- Lateral responses, K = 1/4. Dead band = ±1 degree. Other conditions are as given for figure 5. Motion of the gyro reference is also shown.

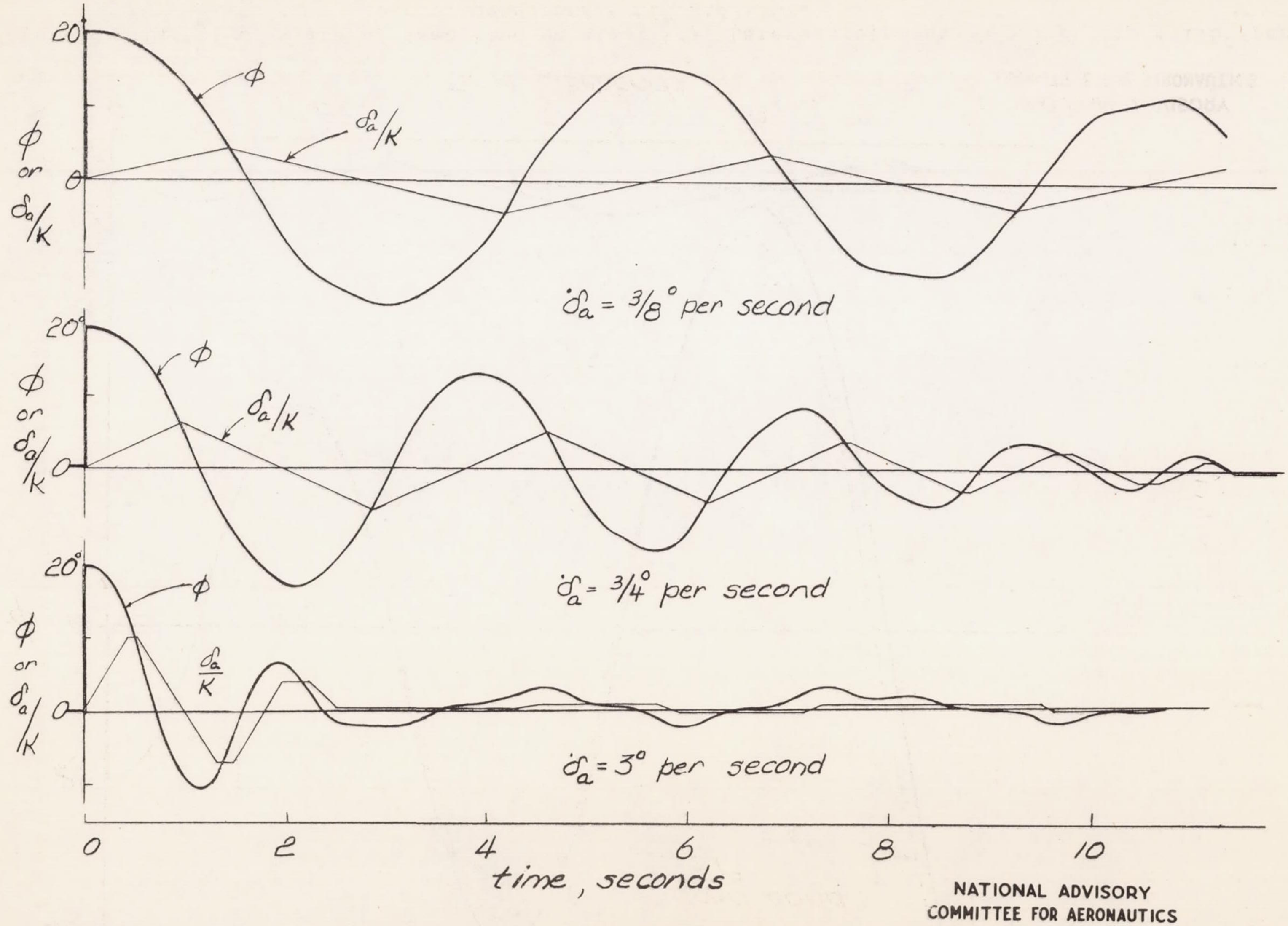
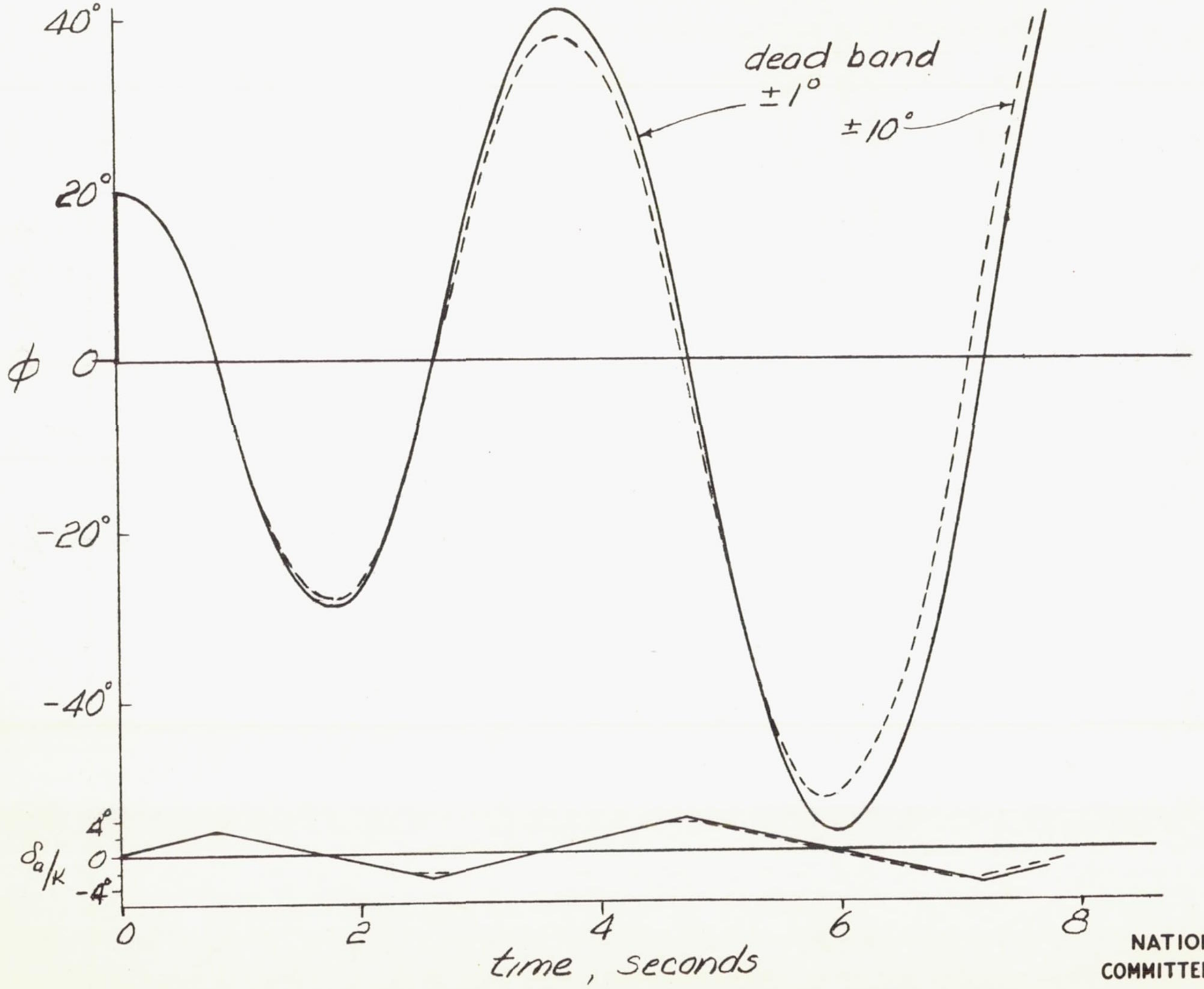



Figure 7.- Lateral responses, K = 1/8. Other conditions as in figure 5.




NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 8.- Effect of width of dead band on stability. Lateral response, K = 1/2; $'\delta_a$ = 1.5° per second; dead band = ±1° and ±10°.

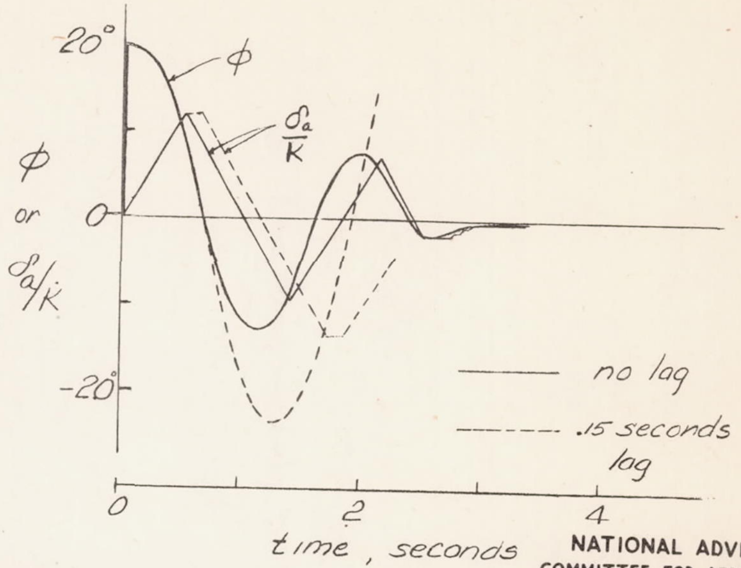



Figure 9.-Lateral motions, K = 1/8 and $\dot{\delta}_a$ = 3° per second. Effect of servo coast causing 0.11° of aileron motion after entering dead band, and of lag. V = 850 feet per second.

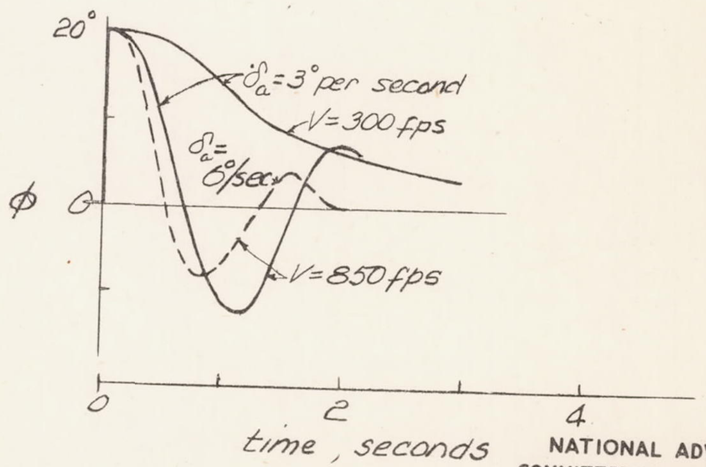
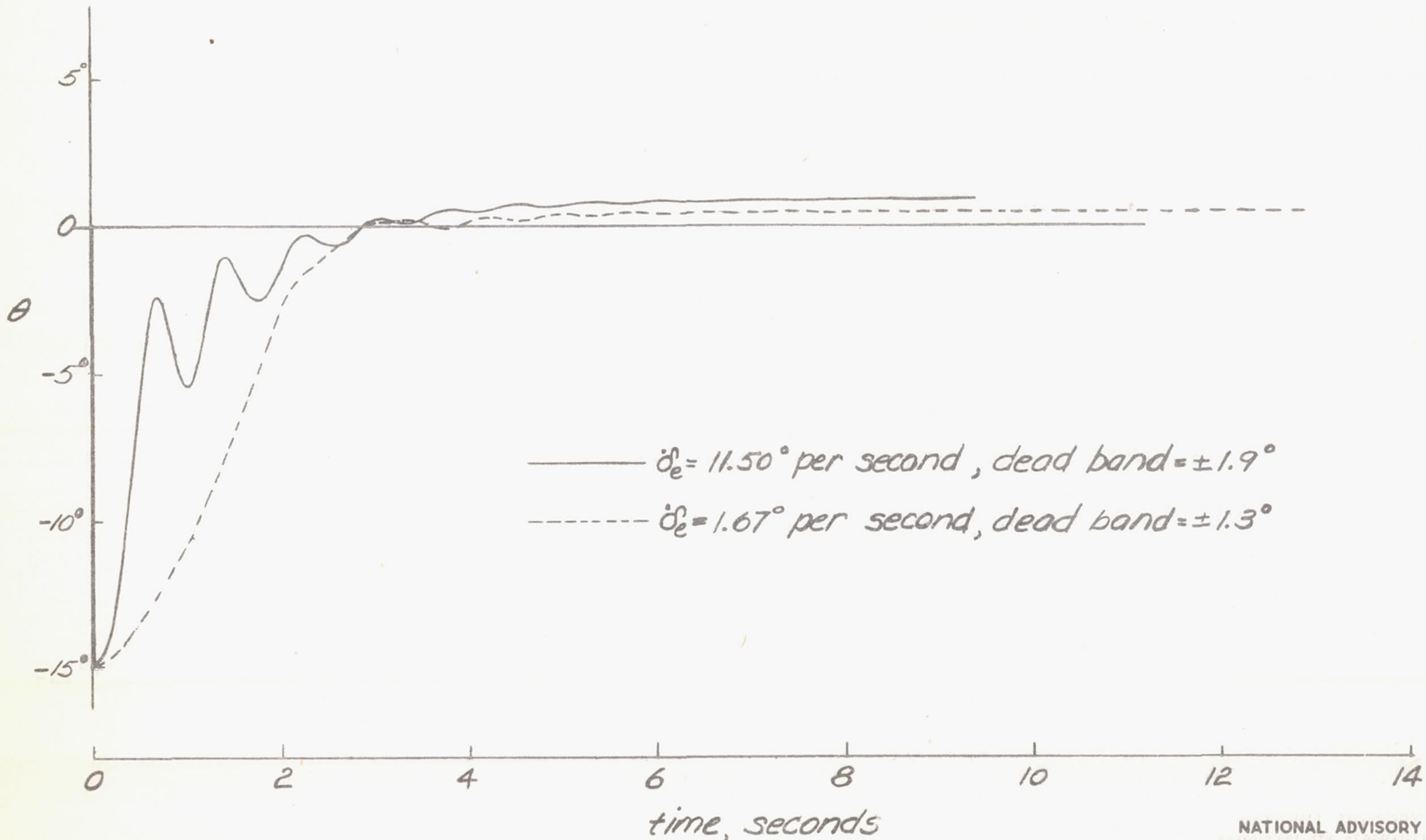



Figure 10.- Lateral motions, K = 1/12. Lag = 0.15 seconds.




NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 11.- Longitudinal motions, K=1/2. No lag or coast included V=850 feet per second

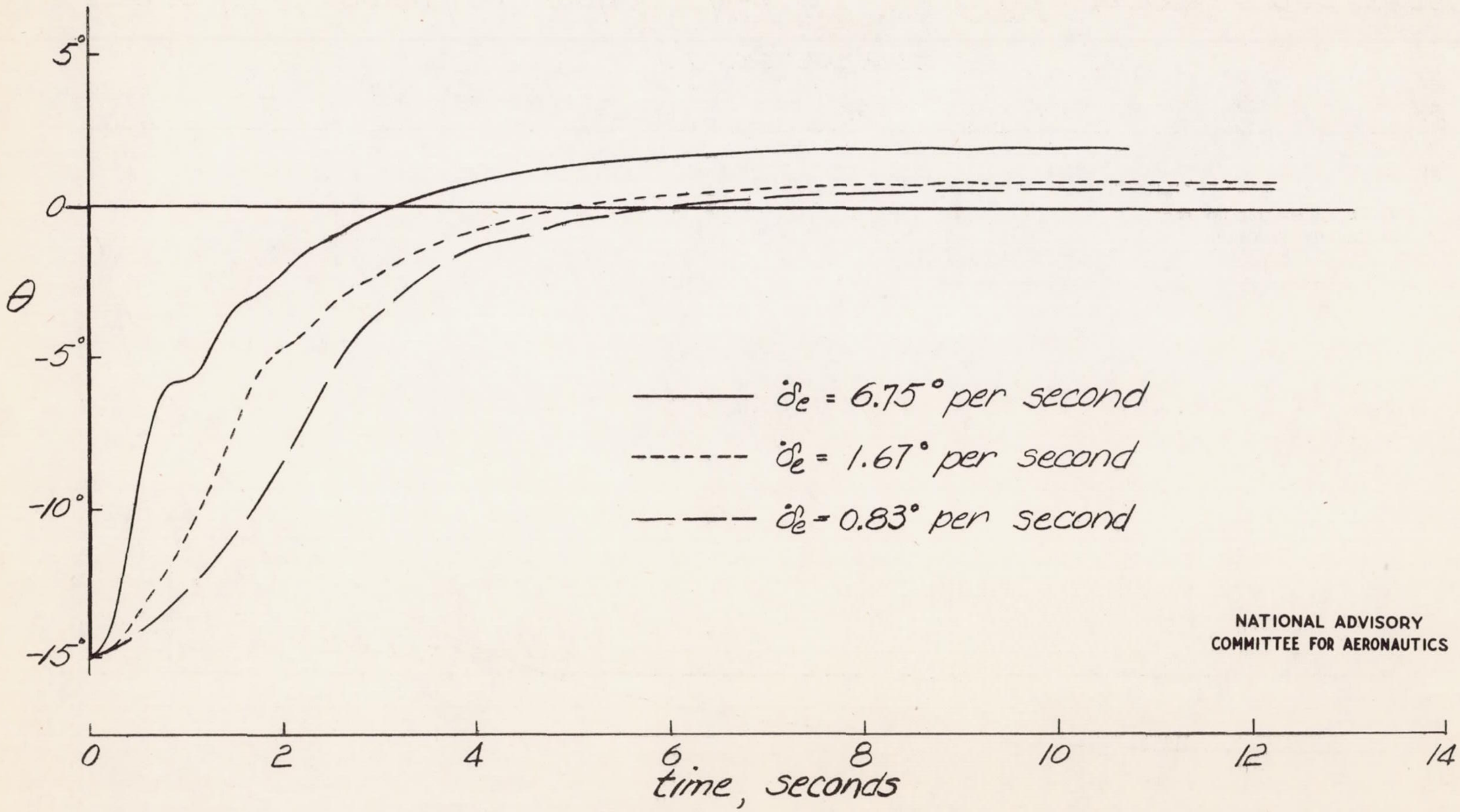





Figure 12.-Longitudinal motions, K=1/4. Dead bands vary from ±2.5° to ±1.3°. V=850 feet per second.

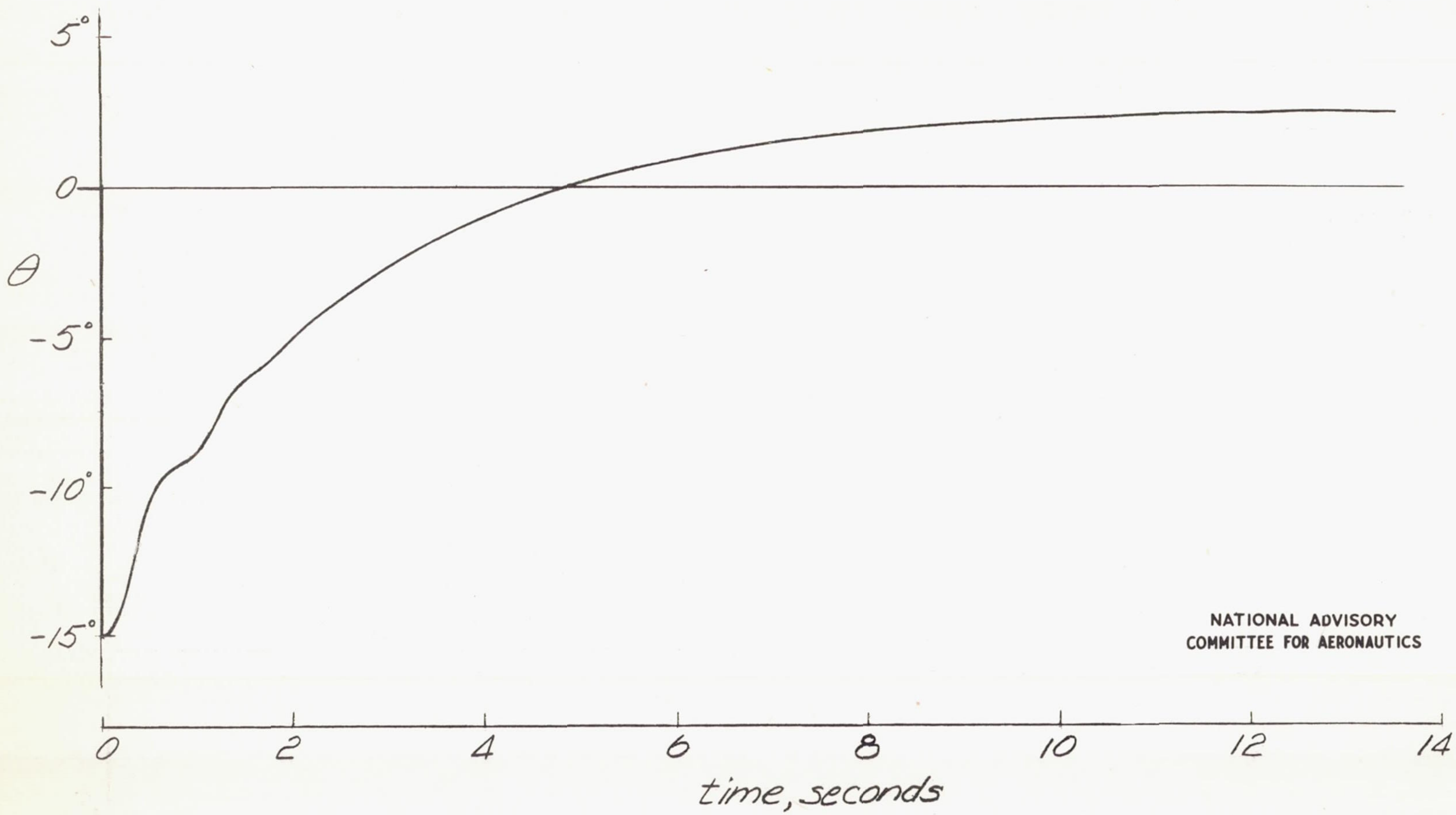
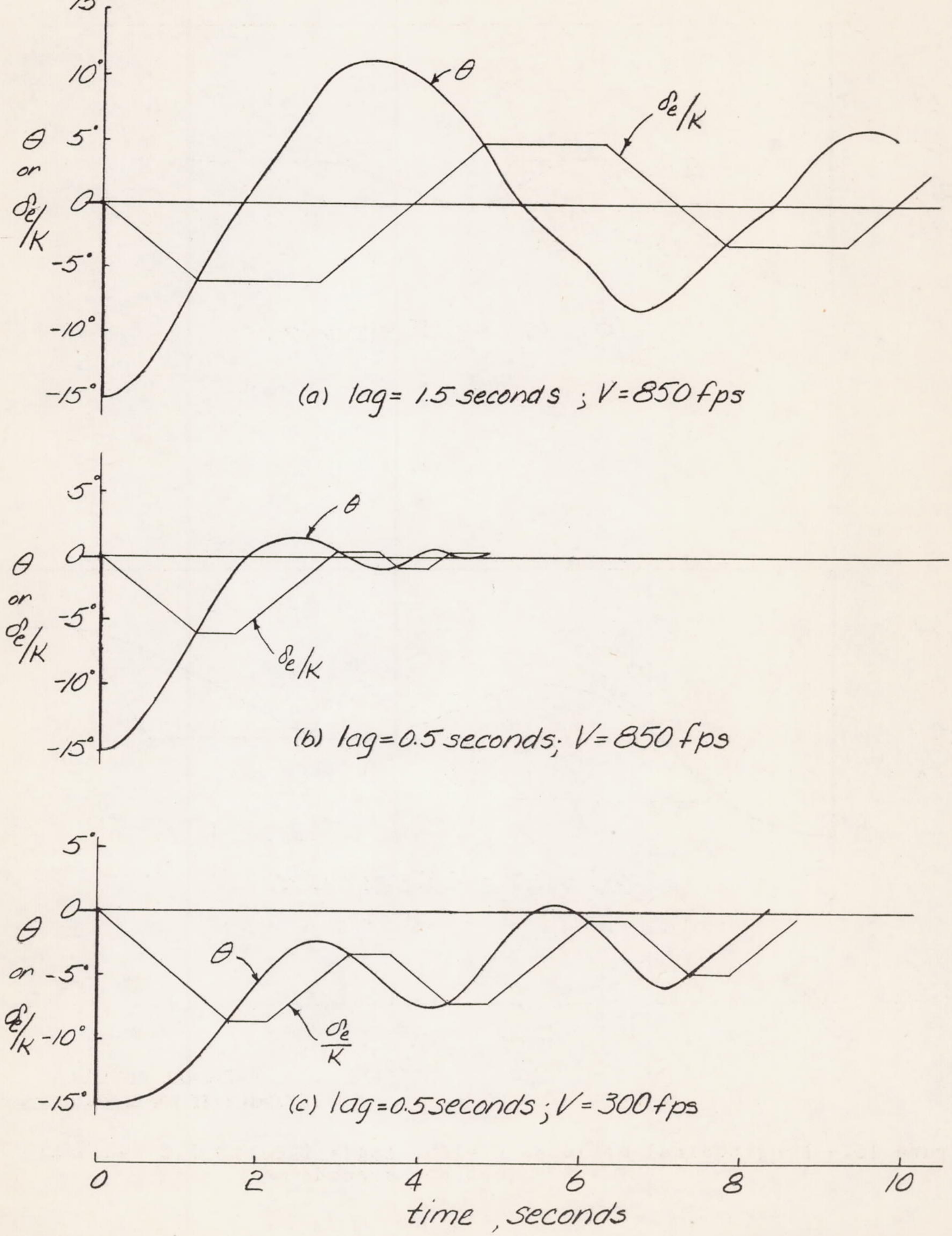
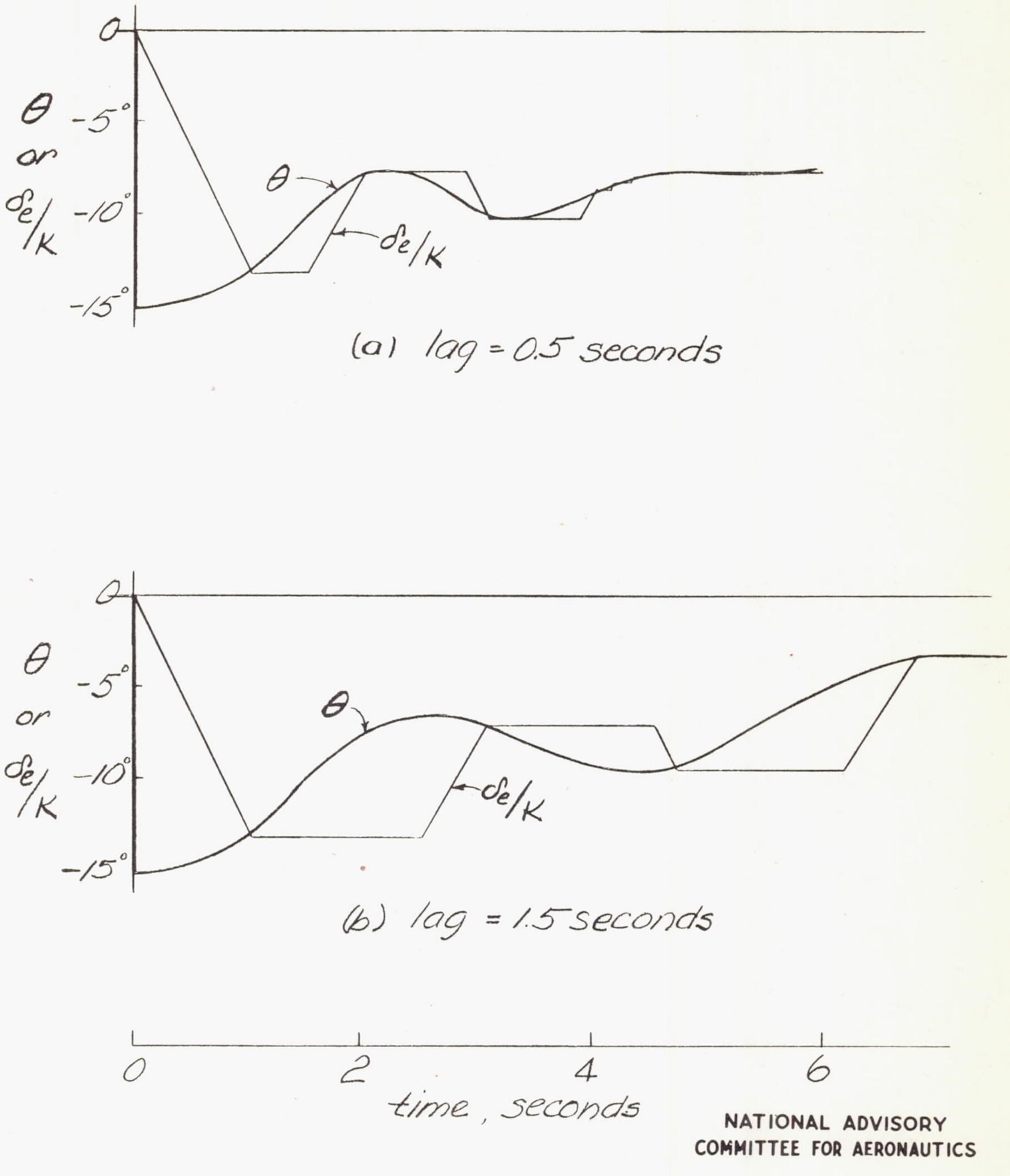



Figure 13. - Longitudinal motion, K = 1/8.; $\dot{\delta}_e$ = 6.75 degrees per second, dead band ±3°. V = 850 feet per second.




NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 14.- Longitudinal motions, K = 1/2, showing the effect of lag.




NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Figure 15.- Longitudinal motions, K =1/5; lag = 0.5 and 1.5 seconds; V = 300 feet per second.

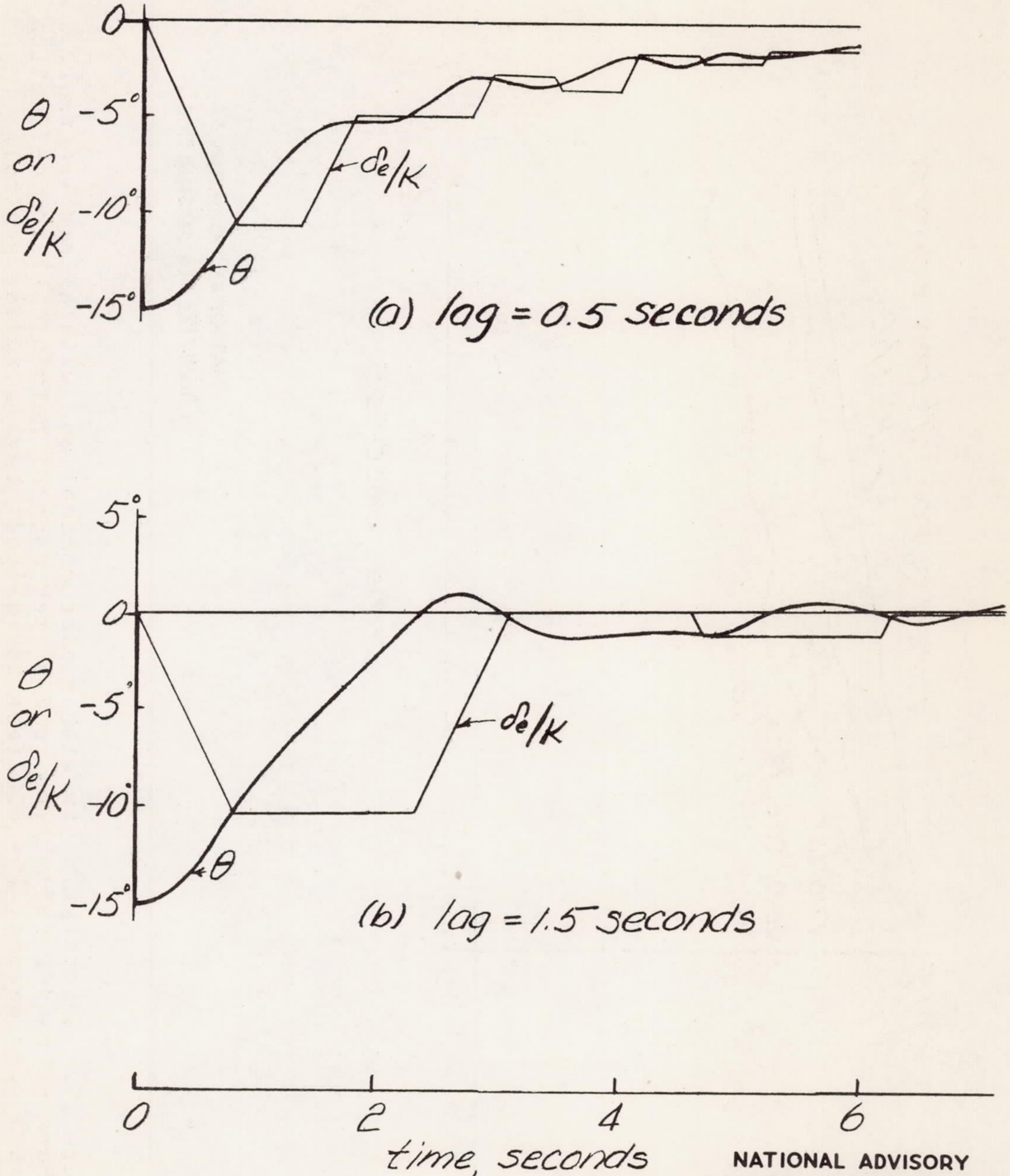
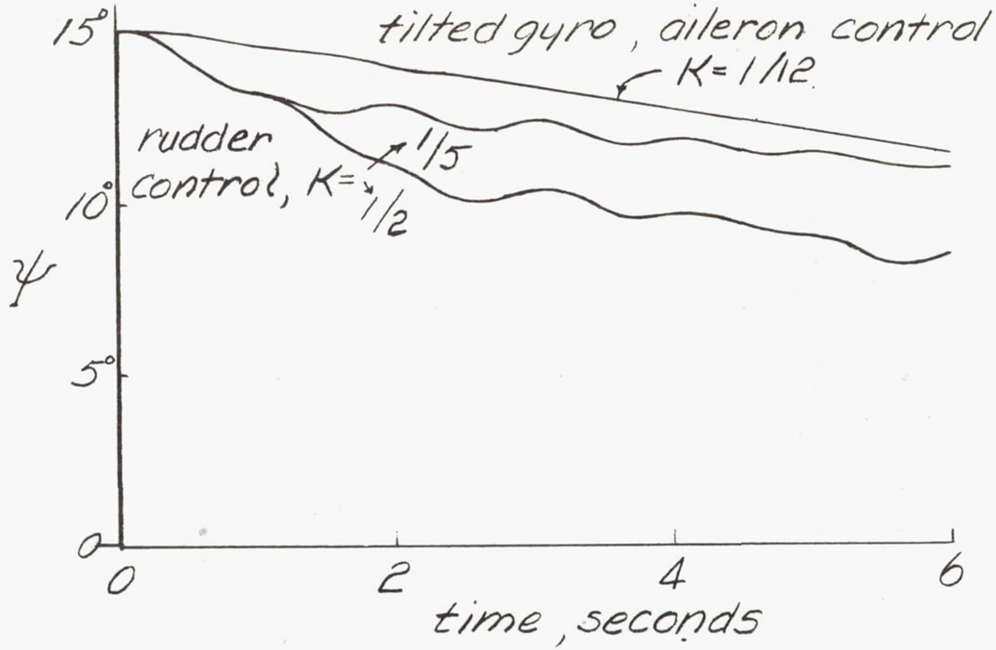





Figure 16.- Longitudinal motion, K = 1/5; lag = 0.5 and 1.5 seconds; V = 850 feet per second.




NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 17.- Recovery in yaw with rudder control and with aileron control. Rate of rudder deflection is 2.5° per second; rate of aileron deflection is 3° per second. V = 850 feet per second. Lag in rudder control, 0.5 seconds.

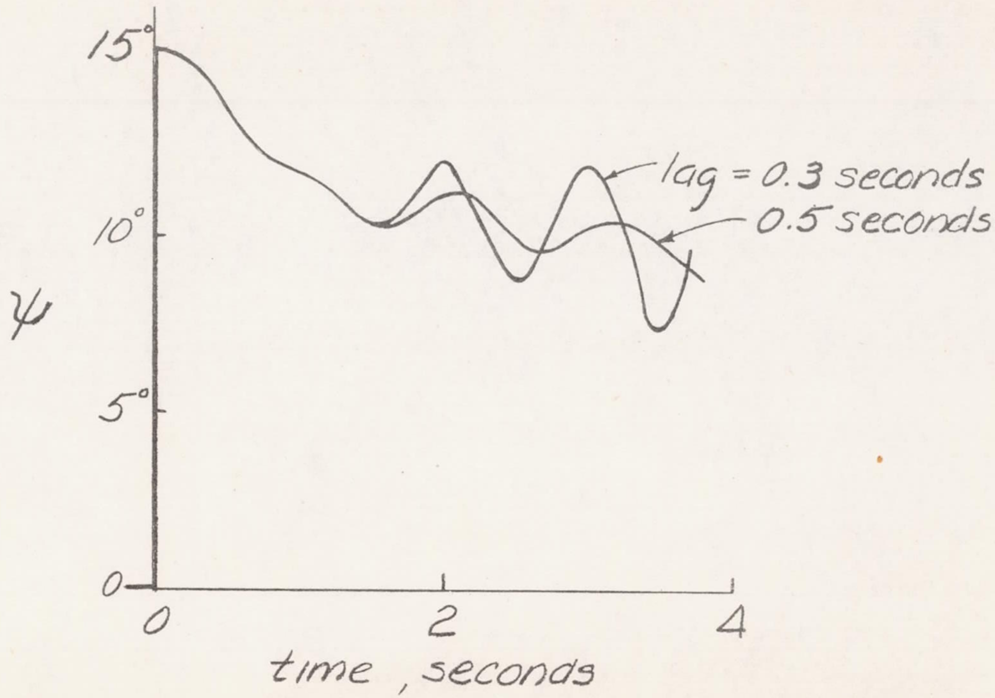







Figure 18.- Yawing motion with rudder control, K = 1/2; $\dot{\delta}_r$ = 5° per second.
V = 850 feet per second.